Uncertainties in nuclear transition matrix elements for neutrinoless $\beta\beta$ decay within the PHFB model

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The nuclear transition matrix elements $M^{(0\nu)}$ for the neutrinoless double beta decay of ^{94,96}Zr, ^{98,100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes in the case of $0^+ \to 0^+$ transition are calculated using the PHFB wave functions, which are eigenvectors of four different parameterizations of a Hamiltonian with pairing plus multipolar effective two-body interaction. Employing two (three) different parameterizations of Jastrow-type short range correlations, a set of eight (twelve) different nuclear transition matrix elements $M^{(0\nu)}$ is built for each decay, whose averages in conjunction with their standard deviations provide an estimate of the model uncertainties.

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I. INTRODUCTION

Ascertaining the mass and nature of neutrinos requires the analysis of observational data obtained from three complementary experiments, namely single-β decay, neutrino oscillation and neutrinoless double beta $(\beta\beta)_{0\nu}$ decay. In any gauge theoretical model with spontaneous symmetry breaking, the observation of $(\beta\beta)_{0\nu}$ decay implies non-zero mass of Majorana neutrinos at the weak scale independent of the underlying mechanisms [1, 2]. The varied scope and far reaching nature of experimental as well as theoretical studies devoted to $(\beta\beta)_{0\nu}$ decay over the past decades have been excellently reviewed by Avignone *et al.* [3] and references there in.

The observed experimental limits on the half-life $T^{0\nu}_{1/2}$ of $(\beta\beta)_{0\nu}$ decay have already provided stringent limits on the associated gauge theoretical parameters [4]. The reliability of extracted gauge theoretical parameters depends on the accuracy of nuclear transition matrix elements (NTMEs). For a given transition, different NTMEs are obtained employing distinct nuclear models, and for a given model, they also depend on the model space and effective two-body interaction selected. Other uncertainties are related with the inclusion of pseudoscalar and weak magnetism terms in the Fermi, Gamow-Teller and tensorial NTMEs [5, 6], finite size as well as short range correlations [7–10], and the use of two effective values of the axial-vector coupling constant g_A.

The spread between the calculated NTMEs provides a measure of the theoretical uncertainty [11]. In the case of the well studied ^{76}Ge isotope, it was observed that the calculated decay rates differ by a factor of 6–7. The effective neutrino mass $\langle m_\nu \rangle$ is inversely proportional to the square root of $T^{0\nu}_{1/2}$. Hence, the uncertainty in the effective neutrino mass is about 2 to 3. For example, from the experimental limit $T^{0\nu}_{1/2} > 1.6 \times 10^{25}$ yr [12], the upper limits on $\langle m_\nu \rangle$ range between 0.4 eV and 1.0 eV, depending on the NTME [13–15]. If the $(\beta\beta)_{0\nu}$ decay were observed in several nuclei, the comparison of calculated ratios of the corresponding NTMEs-squared and the ratios of half-lives could also test the validity of nuclear structure calculations in a model independent way [16].

Rodin *et al.* [17] have estimated the theoretical uncertainty employing two models, the QRPA and RQRPA, with three sets of basis states and three realistic two-body effective interactions. Different strategies to remove the sensitivity of QRPA calculations on the model parameters have been proposed [18, 19]. Further studies on uncertainties in NTMEs due to short range correlations using the unitary correlation operator method (UCOM) [7] and self-consistent coupled cluster method (CCM) [8] have been also carried out by Faessler and coworkers.

Up to now, the QRPA model and its extensions have been the most successful models in correlating the single-β GT strengths and half-lives of $(\beta^-\beta^-)_{2\nu}$ decay and the first in explaining the observed suppression of $M_{2\nu}$ [20, 21]. Nonetheless, the large scale shell model (LSSM) calculations of Strasbourg-Madrid group are quite promising [22]. Deformation has been included at various levels of approximation in the QRPA formalism [23–25]. Recently, the effects of pairing and quadrupolar correlations on the NTMEs of $(\beta^-\beta^-)_{0\nu}$ decay have also been studied in the interacting shell model (ISM) [9, 26] and the projected-Hartree-Fock-Bogoliubov (PHFB) model [27, 28].

The PHFB model, in conjunction with pairing plus quadrupole-quadrupole (PQQ) [29] interaction has been successful in the study of the $0^+ \to 0^+$ transition of $(\beta^-\beta^-)_{2\nu}$ decay, where it was possible to describe the lowest excited states of the parent and daughter nuclei along with their electromagnetic transition strengths, as well as to reproduce their measured $\beta\beta$ decay rates [30, 31]. The PHFB model is unique in allowing the description of the $\beta\beta$ decay in medium and heavy mass nuclei by projecting a set of states with good angular momentum, while treating the pairing and deformation

degrees of freedom simultaneously and on equal footing. On the other hand, in the present version of the PHFB model, the structure of the intermediate odd Z-odd N nuclei and hence, the single β decay rates and the distribution of GT strength can not be studied. Notwithstanding this limitation, it is a convenient choice to examine the explicit role of deformation on the NTMEs. In the study of $\beta^-\beta^-$ decay, there are four noteworthy observations in connection with deformation effects [27, 28], namely:

(i) There exists an inverse correlation between the quadrupole deformation and the size of NTMEs $M_{2\nu}$, $M^{(0\nu)}$ and $M_N^{(0\nu)}$.

(ii) The NTMEs are usually large in the absence of quadrupolar correlations; they are almost constant for small admixture of the QQ interaction and substantially suppressed in deformed nuclei.

(iii) In agreement with the observations made by Šimkovic *et al.* [32], the NTMEs have a well defined maximum when the deformation of parent and daughter nuclei are similar, and they are quite suppressed when the difference in the deformation is large.

(iv) The deformation effects are of equal importance in case of $(\beta^-\beta^-)_{2\nu}$ and $(\beta^-\beta^-)_{0\nu}$ decay.

In earlier works, we have calculated NTMEs $M_{2\nu}$ for the $(\beta^-\beta^-)_{2\nu}$ [30, 31] and $M^{(0\nu)}$ for the $(\beta^-\beta^-)_{0\nu}$ decay [27] with the PQQ effective interaction [29], and the effect of hexadecapolar correlations (HH) [28] on the calculated spectroscopic properties and $(\beta^-\beta^-)$ decay rates has been studied. In the present work, we employ two different parameterizations of the QQ interaction, with and without the HH correlations. Further, the NTMEs $M^{(0\nu)}$ are calculated with three different parametrizations of Jastrow type of SRC employing the four sets of wave functions. The twelve NTMEs provide a reasonable sample for estimating the associated uncertainties. In Sec II, the PHFB formalism employed to describe the $(\beta^-\beta^-)_{0\nu}$ decay with the inclusion of the finite size of the nucleons and short range correlations is shortly reviewed. In Sec. III, the four different parameterizations of the pairing plus multipole Hamiltonian are introduced, the calculated NTMEs vis-a-vis their radial evolution are analyzed, and their average values as well as standard deviations are estimated. Subsequently, the latter are employed to obtain upper limits on the effective mass of light Majorana neutrinos. Conclusions are given in Sec. IV.

II. THEORETICAL FORMALISM

In the Majorana neutrino mass mechanism, the inverse half-life of the $(\beta^-\beta^-)_{0\nu}$ decay due to the exchange of light neutrinos for the $0^+ \to 0^+$ transition is given by [13, 33, 34]

$$\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 G_{01} |M_{GT}^{(0\nu)} - M_F^{(0\nu)}|^2, \tag{1}$$

where the NTMEs $M_k^{(0\nu)}$ are given by

$$M_k^{(0\nu)} = \sum_{n,m} \left\langle 0_F^+ \left\| O_{k,nm} \tau_n^+ \tau_m^+ \right\| 0_I^+ \right\rangle, \tag{2}$$

with

$$O_F = \left(\frac{g_V}{g_A}\right)^2 H(r_{12}), \;\; O_{GT} = \sigma_1 \cdot \sigma_2 \, H(r_{12}) \tag{3}$$

and

$$H(r_{12}) = \frac{R\phi(\overline{A} r_{12})}{r_{12}}. \tag{4}$$

The origin of the neutrino potential $H(r_{12})$ is due to the exchange of light Majorana neutrinos between nucleons being considered as point particles. To take the finite size of nucleons into account, neutrino potential $H(r_{12})$ is folded with a dipole form factor and rewritten as

$$H\left(r_{12}\right) = \frac{4\pi R}{(2\pi)^3} \int d^3 q \frac{\exp\left(i \mathbf{q} \cdot \mathbf{r_{12}}\right)}{q\left(q + \overline{A}\right)} \left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)^4, \tag{5}$$

where

$$\overline{A} = \langle E_N \rangle - \frac{1}{2}\left(E_I + E_F\right). \tag{6}$$

and the cutoff momentum Λ= 850 MeV [27].

The short range correlations (SRC) are produced by the repulsive nucleon-nucleon potential generated through the exchange of ρ and ω mesons. They have been included in the calculations of $M^{(0\nu)}$ for the $(\beta^-\beta^-)_{0\nu}$ decay through the phenomenological Jastrow type of correlations with Miller-Spenser parametrization [35], effective operators [36], exchange of ω-meson [37], UCOM [7, 38] and self-consistent CCM [8]. It has been observed that the effects due to the Jastrow type of correlations with Miller-Spenser parametrization are usually strong [36], where as the UCOM and self-consistent CCM have weak effects. Further, Šimkovic *et al.* [8] have shown that it is possible to parametrize the SRC effects of Argonne V18 and CD-Bonn two nucleon potentials by the Jastrow type of correlations within a few percent accuracy. Explicitly, the effects due to the SRC can be incorporated in the calculation of $M^{(0\nu)}$ through the prescription

$$O_k \to f O_k f, \tag{7}$$

with

$$f(r) = 1 - ce^{-ar^2}(1 - br^2), \tag{8}$$

where a = 1.1, 1.59 and 1.52 fm^{-2}, b = 0.68, 1.45 and 1.88 fm^{-2} and c = 1.0, 0.92 and 0.46 for Miller-Spencer,

Argonne V18 and CD-Bonn NN potentials, respectively. In the next section, the NTMEs $M^{(0\nu)}$ are calculated in the PHFB model by employing these three sets of parameters for the SRC, denoted as SRC1, SRC2 and SRC3, respectively.

The three functions $f(r)$ are plotted in Fig. 1. They

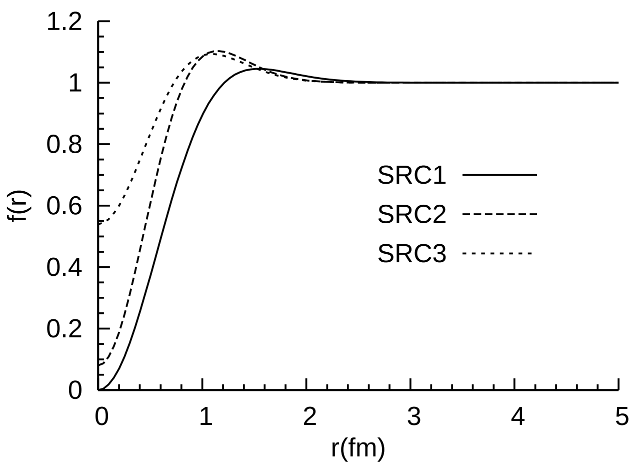

FIG. 1: Radial dependence of $f(r)$ for the three different parameterizations of the SRC.

have similar forms, but differ in its value at the origin, and at the position of its maximum, which lies at 1.54, 1.15 and 1.09 fm for SRC1, SRC2 and SRC3, respectively. They have clear influence on the radial evolution of the $(\beta^-\beta^-)_{0\nu}$ decay matrix elements discussed below.

The calculation of $M^{(0\nu)}$ in the PHFB model has been discussed in Ref. [27] and one obtains the following expression for NTMEs $M_k^{(0\nu)}$ of $(\beta^-\beta^-)_{0\nu}$ decay

$$\begin{aligned} M_k^{(0\nu)} &= \left[n^{Ji=0} n^{J_f=0}\right]^{-1/2} \\ &\times \int_0^\pi n_{(Z,N),(Z+2,N-2)}(\theta) \sum_{\alpha\beta\gamma\delta} \left(\alpha\beta\left|O_k\right|\gamma\delta\right) \\ &\times \sum_{\varepsilon\eta} \frac{\left(f^{(\pi)*}_{Z+2,N-2}\right)_{\varepsilon\beta}}{\left[\left(1+F^{(\pi)}_{Z,N}(\theta) f^{(\pi)*}_{Z+2,N-2}\right)\right]_{\varepsilon\alpha}} \\ &\times \frac{\left(F^{(\nu)*}_{Z,N}\right)_{\eta\delta}}{\left[\left(1+F^{(\nu)}_{Z,N}(\theta) f^{(\nu)*}_{Z+2,N-2}\right)\right]_{\gamma\eta}} sin\theta d\theta, \end{aligned} \quad (9)$$

where

$$\begin{aligned} n^J &= \int_0^\pi \left[det\left(1+F^{(\pi)} f^{(\pi)^\dagger}\right)\right]^{1/2} \\ &\times \left[det\left(1+F^{(\nu)} f^{(\nu)^\dagger}\right)\right]^{1/2} d^J_{00}(\theta) sin(\theta) d\theta, \end{aligned} \quad (10)$$

and

$$\begin{aligned} n_{(Z,N),(Z+2,N-2)}(\theta) &= \left[det\left(1+F^{(\nu)}_{Z,N} f^{(\nu)^\dagger}_{Z+2,N-2}\right)\right]^{1/2} \\ &\times \left[det\left(1+F^{(\pi)}_{Z,N} f^{(\pi)^\dagger}_{Z+2,N-2}\right)\right]^{1/2} \end{aligned} \quad (11)$$

The $\pi(\nu)$ represents the proton (neutron) of nuclei involved in the $(\beta^-\beta^-)_{0\nu}$ decay process. The matrices $f_{Z,N}$ and $F_{Z,N}(\theta)$ are given by

$$f_{Z,N} = \sum_i C_{ij_\alpha,m_\alpha} C_{ij_\beta,m_\beta} \left(v_{im_\alpha}/u_{im_\alpha}\right) \delta_{m_\alpha,-m_\beta}, \quad (12)$$

$$F_{Z,N}(\theta) = \sum_{m'_\alpha m'_\beta} d^{j_\alpha}_{m_\alpha,m'_\alpha}(\theta) d^{j_\beta}_{m_\beta,m'_\beta}(\theta) f_{j_\alpha m'_\alpha, j_\beta m'_\beta}. \quad (13)$$

The extra factor 1/4 in the Eq. (28) of Ref. [27] should not be there.

III. RESULTS AND DISCUSSIONS

The model space, single particle energies (SPE's), parameters of the PQQ type of effective two-body interaction and the method to fix them have been already given in Refs. [27, 30, 31]. Presently, we use the effective Hamiltonian written as [28]

$$H = H_{sp} + V(P) + V(QQ) + V(HH), \quad (14)$$

where H_{sp}, $V(P)$, $V(QQ)$ and $V(HH)$ denote the single particle Hamiltonian, the pairing, quadrupole-quadrupole and hexadecapole-hexadecapole parts of the effective two-body interaction, respectively. The quadrupole-quadrupole part of the effective two-body interaction $V(QQ)$ has three terms, namely the proton-proton, the neutron-neutron and the proton-neutron ones, whose coefficients are denoted by χ_{2pp}, χ_{2nn} and χ_{2pn}, respectively. In Refs. [27, 30, 31], the strengths of the like particle components of the QQ interaction were taken as $\chi_{2pp} = \chi_{2nn} = 0.0105$ MeV b^{-4}, where b is the oscillator parameter. The strength of proton-neutron component of the QQ interaction χ_{2pn} was varied so as to fit the experimental excitation energy of the 2^+ state, E_{2^+}. In the present work, we also employ an alternative isoscalar parametrization of the quadrupole-quadrupole interaction, by taking $\chi_{2pp} = \chi_{2nn} = \chi_{2pn}/2$. In this case, the three parameters are varied together to fit E_{2^+}. We will refer to these two parameterizations of the quadrupole-quadrupole interaction as $PQQ1$ and $PQQ2$.

Employing either method, the experimental excitation energies of 2^+ state E_{2^+} [39] can be reproduced within about 2% accuracy. The maximum change in E_{4^+} and E_{6^+} energies with respect to $PQQ1$ interaction [30, 31] is about 5% and 18%, respectively. The reduced $B(E2{:}0^+ \to 2^+)$ transition probabilities, deformation parameters β_2, static quadrupole moments $Q(2^+)$

and gyromagnetic factors $g(2^+)$ are in an overall agreement with the experimental data [40, 41] for both the parametrizations. In the case of $PQQ2$ parametrization, the maximum change in the calculated NTMEs $M_{2\nu}$ for the $0^+ \to 0^+$ transition with respect to $PQQ1$ parametrization is about 21% but for ^{94}Zr isotope.

The HH part of the effective interaction $V(HH)$ is given as [28]

$$V(HH) = -\left(\frac{\chi_4}{2}\right) \sum_{\alpha\beta\gamma\delta} \sum_{\nu} (-1)^{\nu} \langle\alpha|q_{4\nu}|\gamma\rangle \times \langle\beta|q_{4-\nu}|\delta\rangle \; a_{\alpha}^{\dagger} a_{\beta}^{\dagger} \; a_{\delta} \; a_{\gamma}, \tag{15}$$

with $q_{4\nu} = r^4 Y_{4\nu}(\theta,\phi)$. The relative magnitudes of the parameters of the HH part of the two body interaction are calculated from a relation suggested by Bohr and Mottelson [42]. The approximate magnitude of these constants for isospin $T = 0$ is given by

$$\chi_\lambda = \frac{4\pi}{2\lambda+1} \frac{m\omega_0^2}{A\,\langle r^{2\lambda-2}\rangle} \qquad for\, \lambda = 1,2,3,4\cdots \tag{16}$$

and the parameters for the $T = 1$ case are approximately half of their $T = 0$ counterparts. Presently, the value of $\chi_4 = 0.2442\ \chi_2 A^{-2/3} b^{-4}$ for $T = 1$, which is exactly half of the $T = 0$ case.

We refer to the calculations which include the hexadecapolar term HH as $PQQHH$. We end up with four different parameterizations of the effective two-body interaction, namely $PQQ1$, $PQQHH1$, $PQQ2$ and $PQQHH2$.

A. SRC and radial evolutions of NTMEs

In Table I, the NTMEs $M^{(0\nu)}$ evaluated using the HFB wave functions in conjunction with $PQQ1$, $PQQHH1$, $PQQ2$, $PQQHH2$ interactions and three different parametrizations of the Jastrow type of SRC for the nuclei ^{94,96}Zr, ^{98,100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd are displayed. The average energy denominator $\overline{A}$ has been taken as $\overline{A} = 1.12A^{1/2}$ MeV following Haxton's prescription [13]. The NTMEs are calculated in the the approximations of point nucleons (P - 2nd and 3rd columns), finite size of nucleons (F - 7th column), point nucleons with SRC (P+S - 4th to 6th columns), and finite size plus SRC (F+S - last three columns). To obtain additional information on the stability of the estimations of NTMEs $M^{(0\nu)}$, they are also calculated for $\overline{A}/2$ in the energy denominator in the case of point nucleons, given in the column 3.

We present the relative changes in NTMEs $M^{(0\nu)}$ (in %) due to the different approximations in Table II. In each row, i.e. for each set of wave functions, the reference NTMEs $M^{(0\nu)}$ are those calculated for point nucleons without SRC, given in the second column of Table I. It can be observed that the relative change in NTMEs $M^{(0\nu)}$, when the energy denominator is taken as $\overline{A}/2$ instead of $\overline{A}$, is of the order of 10 %. It confirms that the dependence of NTMEs on average excitation energy $\overline{A}$ is small for the $(\beta^-\beta^-)_{0\nu}$ decay and the validity of the closure approximation is quite satisfactory.

The variation in $M^{(0\nu)}$ due to the different parameterizations of the Hamiltonians (presented in the different rows) lies between 20–25%. It is noticed in general but for ^{128}Te isotope that the NTMEs evaluated for both parameterizations of the quadrupolar interaction are quite close. The inclusion of the hexadecapolar term tends to reduce them by amounts which strongly depend on the specific nuclei.

The inclusion of SRC in the approximation of point nucleons (P+S) induces an extra quenching in the NTMEs $M^{(0\nu)}$, which can be of the order of 18–23% for SRC1, to negligible for SRC3. The dipole form factor (F) always reduces the NTMEs by 12–15% in comparision to the point particle case. Adding SRC (F+S) can further reduce the transition matrix elements, for SRC1, or slightly enhance them, partially compensating the effect of the dipole form factor. It is interesting to note that the effect of F-SRC2 is almost negligible, i.e., nearly the same as F.

The radial evolution of $M^{(0\nu)}$ has been studied in the QRPA by Šimkovic *et al.* [7] and in the ISM by Menéndez *et al.* [43] by defining

$$M^{(0\nu)} = \int C^{(0\nu)}(r)\, dr. \tag{17}$$

In both QRPA and ISM calculations, it has been established that the contributions of decaying pairs coupled to $J = 0$ and $J > 0$ almost cancel beyond $r \approx 3$ fm and the magnitude of $C^{(0\nu)}$ for all nuclei undergoing $(\beta^-\beta^-)_{0\nu}$ decay are the maximum about the internucleon distance $r \approx 1$ fm. In Fig. 2, we plot the radial dependence of $C^{(0\nu)}$ due to $PQQ1$ parametrization of the effective two body interaction for six nuclei, namely ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd. The radial evolution of $M^{(0\nu)}$ is studied for eight cases, namely P, P+SRC1, P+SRC2, P+SRC3, F, F+SRC1, F+SRC2 and F+SRC3. In addition, the effects due to the finite size and SRC are made more transparent in Fig. 3 by plotting them for different combinations of P, F and SRC. In case of point nucleons, it is noticed that the $C^{(0\nu)}$ are peaked at $r = 1.0$ fm and with the addition of SRC1, the peak shifts to 1.25 fm. However, the magnitude of $C^{(0\nu)}$ are increased for SRC2 and SRC3 with unchanging peak position. In the case of FNS, the $C^{(0\nu)}$ are peaked at $r = 1.25$ fm, which remains unchanged with the inclusion of SRC1, SRC2 and SRC3. However, the magnitudes of $C^{(0\nu)}$ change in the latter three cases. The above observations also remain valid with the other three parametrizations of the effective two-body interaction.

TABLE I: Calculated NTMEs $M^{(0\nu)}$ in the PHFB model with four different parameterizations of the effective two-body interaction, three different parameterizations of SRC, with nucleons taken as point particles (P) or with a dipole form factor (F), for the $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{94,96}Zr, ^{98,100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes.

Nuclei		$M^{(0\nu)}$								
		P		P+S			F	F+S		
		A	$A/2$	SRC1	SRC2	SRC3		SRC1	SRC2	SRC3
^{94}Zr	$PQQ1$	5.4382	5.8729	4.3021	5.0644	5.4097	4.6891	4.0690	4.6639	4.8383
	$PQQHH1$	5.0015	5.3947	3.9472	4.6528	4.9734	4.3069	3.7315	4.2820	4.4441
	$PQQ2$	5.1183	5.5700	4.1781	4.8201	5.1063	4.4912	3.9802	4.4818	4.6259
	$PQQHH2$	4.7626	5.1356	3.7492	4.4266	4.7348	4.0955	3.5424	4.0708	4.2266
^{96}Zr	$PQQ1$	3.9517	4.2741	3.0829	3.6622	3.9257	3.3828	2.9068	3.3590	3.4923
	$PQQHH1$	3.9363	4.2413	3.0330	3.6333	3.9072	3.3459	2.8507	3.3192	3.4578
	$PQQ2$	3.7804	4.0875	2.9450	3.5017	3.7549	3.2335	2.7758	3.2103	3.3385
	$PQQHH2$	3.7035	3.9891	2.8470	3.4158	3.6753	3.1442	2.6745	3.1182	3.2497
^{98}Mo	$PQQ1$	8.7743	9.5345	7.0859	8.2350	8.7484	7.6507	6.7322	7.6297	7.8884
	$PQQHH1$	8.1669	8.8420	6.5385	7.6442	8.1395	7.0846	6.1984	7.0618	7.3114
	$PQQ2$	8.8254	9.5866	7.1202	8.2806	8.7992	7.6907	6.7630	7.6695	7.9307
	$PQQHH2$	8.0911	8.7589	6.4723	7.5712	8.0636	7.0154	6.1344	6.9925	7.2406
^{100}Mo	$PQQ1$	8.5939	9.2939	6.8691	8.0522	8.5763	7.4413	6.5036	7.4282	7.6920
	$PQQHH1$	8.2130	8.8577	6.5174	7.6763	8.1915	7.0822	6.1597	7.0654	7.3248
	$PQQ2$	8.6571	9.3633	6.9212	8.1116	8.6391	7.4972	6.5534	7.4838	7.7493
	$PQQHH2$	7.4186	7.9968	5.8774	6.9312	7.3994	6.3904	5.5520	6.3756	6.6113
^{104}Ru	$PQQ1$	6.2757	6.7734	4.9743	5.8753	6.2705	5.4007	4.6942	5.3989	5.5975
	$PQQHH1$	5.7976	6.2339	4.5484	5.4102	5.7895	4.9596	4.2809	4.9548	5.1454
	$PQQ2$	5.9034	6.3698	4.6777	5.5267	5.8989	5.0789	4.4137	5.0777	5.2647
	$PQQHH2$	5.3786	5.7803	4.2143	5.0176	5.3711	4.5974	3.9648	4.5931	4.7708
^{110}Pd	$PQQ1$	10.1361	11.0441	8.1250	9.5068	10.1167	8.7918	7.6982	8.7783	9.0850
	$PQQHH1$	8.5617	9.2893	6.7742	7.9988	8.5408	7.3694	6.3963	7.3535	7.6262
	$PQQ2$	9.7208	10.5944	7.7929	9.1163	9.7011	8.4328	7.3842	8.4187	8.7128
	$PQQHH2$	9.0246	9.8138	7.1864	8.4447	9.0023	7.7985	6.7982	7.7816	8.0621
^{128}Te	$PQQ1$	4.3415	4.7394	3.4372	4.0474	4.3219	3.7417	3.2499	3.7258	3.8639
	$PQQHH1$	4.8152	5.2111	3.7261	4.4626	4.7931	4.0916	3.4994	4.0740	4.2401
	$PQQ2$	5.1422	5.6212	4.1056	4.8082	5.1233	4.4521	3.8893	4.4374	4.5956
	$PQQHH2$	5.0701	5.5058	3.9637	4.7118	5.0477	4.3351	3.7336	4.3172	4.4860
^{130}Te	$PQQ1$	5.7440	6.3018	4.6613	5.4025	5.7319	5.0177	4.4319	5.0103	5.1753
	$PQQHH1$	4.9231	5.3418	3.8530	4.5817	4.9067	4.2084	3.6277	4.1964	4.3595
	$PQQ2$	5.6568	6.2055	4.5875	5.3192	5.6446	4.9397	4.3610	4.9320	5.0951
	$PQQHH2$	4.9115	5.3304	3.8459	4.5714	4.8951	4.1999	3.6218	4.1879	4.3503
^{150}Nd	$PQQ1$	4.1436	4.5674	3.3937	3.9137	4.1420	3.6355	3.2316	3.6375	3.7514
	$PQQHH1$	3.1506	3.4603	2.5501	2.9650	3.1478	2.7448	2.4208	2.7447	2.8359
	$PQQ2$	4.0499	4.4632	3.3160	3.8249	4.0483	3.5526	3.1574	3.5546	3.6661
	$PQQHH2$	3.2415	3.5638	2.6341	3.0545	3.2392	2.8305	2.5031	2.8311	2.9234

B. Uncertainties in NTMEs

To estimate the uncertainties associated with the NTMEs $M^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay calculated using the PHFB model, we evaluate their mean and the standard deviation, defined as

$$\overline{M}^{(0\nu)} = \frac{\sum_{i=1}^{N} M_i^{(0\nu)}}{N} \tag{18}$$

and

$$\Delta\overline{M}^{(0\nu)} = \frac{1}{\sqrt{N-1}}\left[\sum_{i=1}^{N}\left(\overline{M}^{(0\nu)} - M_i^{(0\nu)}\right)^2\right]^{1/2}. \tag{19}$$

Recently, it has been shown by Šimkovic *et al.* [8] that the phenomenological Jastrow correlations with Miller-Spenser parametrization is a major source of uncertainty. Therefore, it is more appropriate to consider SRC2 or

TABLE II: Maximum and minimum relative change in the NTME $M^{(0\nu)}$ (in %), for all nuclei included in table I, due to the use of a different average energy denominator (second column), the inclusion of three different parameterizations of the SRC (SRC1, SRC2 and SRC3) with point nucleons (third to fifth column), the inclusion of finite size effect (F) (sixth column) and finite size effect plus SRC (F+SRC1, F+SRC2 and F+SRC3 in last three columns). In each row, the results employing one of the four different parameterizations of the effective two-body interaction are displayed.

Parametrizatios	$\overline{A}/2$	P+S			F	F+S		
		SRC1	SRC2	SRC3		SRC1	SRC2	SRC3
$PQQ1$	7.9–10.2	18.1–22.0	5.5–7.3	0.04–0.7	12.3–13.9	22.0–26.4	12.2–15.0	9.5–11.6
$PQQHH1$	7.5–9.8	19.1–22.9	5.9–7.7	0.1–0.7	12.9–15.0	23.2–27.6	12.9–15.7	10.0–12.5
$PQQ2$	7.9–10.2	18.1–22.1	5.6–7.4	0.04–0.7	12.2–14.5	22.0–26.6	12.2–15.1	9.5–11.7
$PQQHH2$	7.4–9.9	18.7–23.1	5.8–7.8	0.1–0.8	12.7–15.1	22.8–27.8	12.7–15.8	9.8–12.2

TABLE III: Average NTMEs $\overline{M}^{(0\nu)}$ and uncertainties $\Delta\overline{M}^{(0\nu)}$ for the $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{94,96}Zr, ^{98,100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes. Both bare and quenched values of g_A are considered.

$\beta^-\beta^-$ emitters	g_A	Case I		Case II	
		$\overline{M}^{(0\nu)}$	$\Delta\overline{M}^{(0\nu)}$	$\overline{M}^{(0\nu)}$	$\Delta\overline{M}^{(0\nu)}$
^{94}Zr	1.254	4.2464	0.3883	4.4542	0.2536
	1.0	4.6382	0.4246	4.8668	0.2759
^{96}Zr	1.254	3.1461	0.2778	3.3181	0.1243
	1.0	3.4481	0.3085	3.6376	0.1424
^{98}Mo	1.254	7.1294	0.6013	7.4656	0.3635
	1.0	7.8398	0.6826	8.2099	0.4358
^{100}Mo	1.254	6.8749	0.6855	7.2163	0.4977
	1.0	7.5660	0.7744	7.9419	0.5769
^{110}Pd	1.254	7.8413	0.8124	8.2273	0.6167
	1.0	8.6120	0.9184	9.0370	0.7128
^{128}Te	1.254	4.0094	0.4194	4.2175	0.3074
	1.0	4.4281	0.4601	4.6571	0.3355
^{130}Te	1.254	4.4458	0.5231	4.6633	0.4269
	1.0	4.9065	0.5837	5.1459	0.4802
^{150}Nd	1.254	3.1048	0.4649	3.2431	0.4434
	1.0	3.4334	0.5181	3.5856	0.4952

SRC3 due to the Argonne V18 and CD-Bonn NN potentials, respectively. Based on these observations, we perform the statistical analysis of two cases. In case I, we calculate the average and variance of twelve NTMEs listed in the last three columns (F+S) of Table I with the bare and quenched values of axial vector coupling constant $g_A = 1.254$ and $g_A = 1.0$, respectively. The average and standard deviations of eight NTMES $M^{(0\nu)}$ due to SRC2 and SRC3 are similarly calculated in the case II. The average NTMEs $\overline{M}^{(0\nu)}$ and standard deviations $\Delta\overline{M}^{(0\nu)}$ are presented in Table III. It is noticed that the exclusion of Miller-Spenser parametrization reduces the uncertainty by about 55% in ^{96}Zr to 4% in ^{150}Nd isotope. In Table IV, we present the average NTMEs $\overline{M}^{(0\nu)}$ of case II along with the recently reported results in ISM by Caurier *et al.* [9], QRPA as well as RQRPA by Šimkovic *et al.* [8] and IBM by Barea and Iachello [44]. In spite of the fact that different model space, two-body interactions and SRC have been used in these models, the spread in the NTMEs turns out to be about a factor of 2.5. Further, we extract upper limits on the effective mass of light neutrinos $\langle m_\nu \rangle$ from the largest observed limits on half-lives $T_{1/2}^{0\nu}$ of $(\beta^-\beta^-)_{0\nu}$ decay using the phase space factors of Boehm and Vogel [45]. It is observed that the extracted limits on $\langle m_\nu \rangle$ for ^{100}Mo and ^{130}Te nuclei are $0.48^{+0.04}_{-0.03} - 0.69^{+0.05}_{-0.05}$ and $0.30^{+0.03}_{-0.02} - 0.42^{+0.04}_{-0.04}$ eV, respectively. In the last column of Table IV, the predicted half-lives of $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{94,96}Zr, ^{98,100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes are given for $\langle m_\nu \rangle = 50$ meV.

IV. CONCLUSIONS

We have studied the $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{94,96}Zr, ^{98,100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes in the light Majorana neutrino mass mechanism using a set of PHFB wave functions. The reliability of wave functions generated with $PQQ1$ and $PQQHH1$ interactions has been tested in previous works by calculating the yrast spectra, reduced $B(E2{:}0^+ \to 2^+)$ transition probabilities, static quadrupole moments $Q(2^+)$ and g-factors $g(2^+)$ of participating nuclei in $\left(\beta^-\beta^-\right)_{2\nu}$ decay as well as $M_{2\nu}$ and comparing them with the available experimental data [30, 31]. An overall agreement between the calculated and observed spectroscopic properties as well as $M_{2\nu}$ suggests that the PHFB wave functions generated by fixing χ_{pn} or χ_{pp} to reproduce the E_{2^+} are reasonably reliable.

In the present work, NTMEs $M^{(0\nu)}$ were calculated employing the PHFB model with four different parameterizations of the pairing plus multipolar type of effective two body interaction and two(three) different parameterizations of the short range correlations. It was found that the NTMEs $M^{(0\nu)}$ change by about 4–14(10–15)%.

The mean and standard deviations were evaluated for the NTMEs $M^{(0\nu)}$ calculated with dipole form fac-

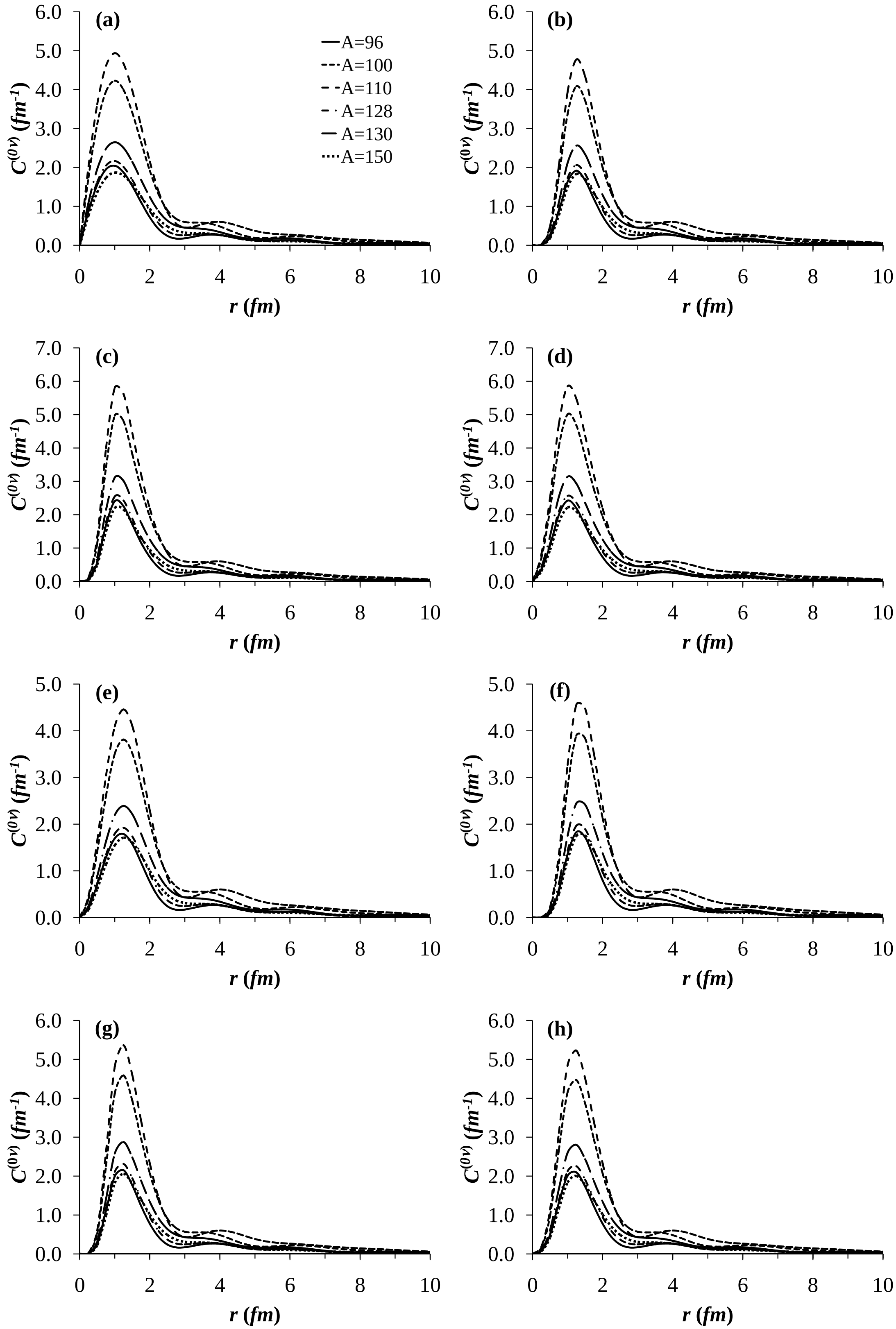

FIG. 2: Radial dependence of $C^{(0\nu)}(r)$ for the $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes. In this Fig., (a), (b), (c) and (d) correspond to P, P+SRC1, P+SRC2 and P+SRC3, respectively. Further, (e), (f), (g) and (h) are for F, F+SRC1, F+SRC2 and F+SRC3, respectively.

TABLE IV: Extracted limits on effective light Majorana neutrino mass $\langle m_\nu \rangle$ and predicted half lives using average NTMEs $\overline{M}^{(0\nu)}$ and uncertainties $\Delta\overline{M}^{(0\nu)}$ for the $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{94,96}Zr, ^{98,100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes.

$\beta^-\beta^-$ emitters	g_A	$\overline{M}^{(0\nu)}$	ISM [9]	(R)QRPA [8]	IBM [44]	G_{01} (10^{-14}y^{-1})	$T^{0\nu}_{1/2}$(yr)	Ref.	$\langle m_\nu \rangle$	$T^{0\nu}_{1/2}$(y) $< m_\nu >= 50\ meV$
^{94}Zr	1.254	4.45±0.25				0.1 684	1.9×10^{19}	[46]	$6.41^{+0.39}_{-0.35}\times 10^2$	$3.13^{+0.39}_{-0.33}\times10^{27}$
	1.0	4.87±0.28							$9.23^{+0.56}_{-0.49}\times 10^2$	$6.48^{+0.80}_{-0.68}\times10^{27}$
^{96}Zr	1.254	3.32±0.12		1.43–2.12		5. 930	1.0×10^{21}	[46]	$20.00^{+0.78}_{-0.72}$	$1.60^{+0.13}_{-0.11}\times10^{26}$
	1.0	3.64±0.14							$28.70^{+1.17}_{-1.08}$	$3.29^{+0.27}_{-0.24}\times10^{26}$
^{98}Mo	1.254	7.47±0.36				0.0018	1.0×10^{14}	[47]	$1.62^{+0.08}_{-0.08}\times 10^6$	$1.06^{+0.11}_{-0.10}\times10^{29}$
	1.0	8.21±0.44							$2.32^{+0.13}_{-0.12}\times 10^6$	$2.16^{+0.25}_{-0.21}\times10^{29}$
^{100}Mo	1.254	7.22±0.50		2.91–5.56	3.732	4. 640	4.6×10^{23}	[48]	$0.48^{+0.04}_{-0.03}$	$4.32^{+0.66}_{-0.54}\times10^{25}$
	1.0	7.94±0.58							$0.69^{+0.05}_{-0.05}$	$8.83^{+1.44}_{-1.15}\times10^{25}$
^{110}Pd	1.254	8.23±0.62				1. 422	6.0×10^{17}	[49]	$6.72^{+0.54}_{-0.47}\times 10^2$	$1.09^{+0.18}_{-0.15}\times10^{26}$
	1.0	9.04±0.71							$9.63^{+0.82}_{-0.70}\times 10^2$	$2.22^{+0.40}_{-0.31}\times10^{26}$
^{128}Te	1.254	4.22±0.31	2.26	3.21–5.65	4.517	0.1849	1.1×10^{23}	[50]	$8.50^{+0.67}_{-0.58}$	$3.18^{+0.52}_{-0.42}\times10^{27}$
	1.0	4.66±0.34							$12.10^{+0.94}_{-0.81}$	$6.44^{+1.04}_{-0.84}\times10^{27}$
^{130}Te	1.254	4.66±0.43	2.04	2.92–5.04	4.059	4. 490	3.0×10^{24}	[51]	$0.30^{+0.03}_{-0.02}$	$1.07^{+0.23}_{-0.17}\times10^{26}$
	1.0	5.15±0.48							$0.42^{+0.04}_{-0.04}$	$2.17^{+0.47}_{-0.35}\times10^{26}$
^{150}Nd	1.254	3.24±0.44			2.321	21.16	1.8×10^{22}	[52]	$2.55^{+0.40}_{-0.31}$	$4.69^{+1.60}_{-1.06}\times10^{25}$
	1.0	3.59±0.50							$3.63^{+0.58}_{-0.44}$	$9.49^{+3.29}_{-2.16}\times10^{25}$

tor and with and without Miller-Spencer parametrization of short range correlations, which were employed to estimate the $(\beta^-\beta^-)_{0\nu}$ decay half-lives $T^{0\nu}_{1/2}$ for both $g_A = 1.254$ and $g_A = 1.0$. The largest standard deviation, interpreted as theoretical uncertainty, turns out to be of the order of 15% in the case of ^{150}Nd isotope. We have also extracted limits on the effective mass of light Majorana neutrinos $\langle m_\nu \rangle$ from the available limits on experimental half-lives $T^{0\nu}_{1/2}$ using average NTMEs $\overline{M}^{(0\nu)}$ calculated in the PHFB model. In the case of ^{130}Te isotope, one obtains the best limit on the effective neutrino mass $\langle m_\nu \rangle < 0.30^{+0.03}_{-0.02} - 0.42^{+0.04}_{-0.04}$ eV from the observed limit on the half-lives $T^{0\nu}_{1/2} > 3.0 \times 10^{24}$ yr of $(\beta^-\beta^-)_{0\nu}$ decay [51].

Note: *Due to an error in one equation, the NTMEs M_F, M_{GT}, $M^{(0\nu)}$, M_{Fh}, M_{GTh} and $M_N^{(0\nu)}$ given in Ref. [27] must be multiplied by a factor of 2. It implies that the limits on the effective light neutrino mass $< m_\nu >$ must be reduced by a factor of 2 whereas the limits on effective heavy neutrino mass $< M_N >$ must be multiplied by a factor of 2. In both cases the limits are twice more stringent.*

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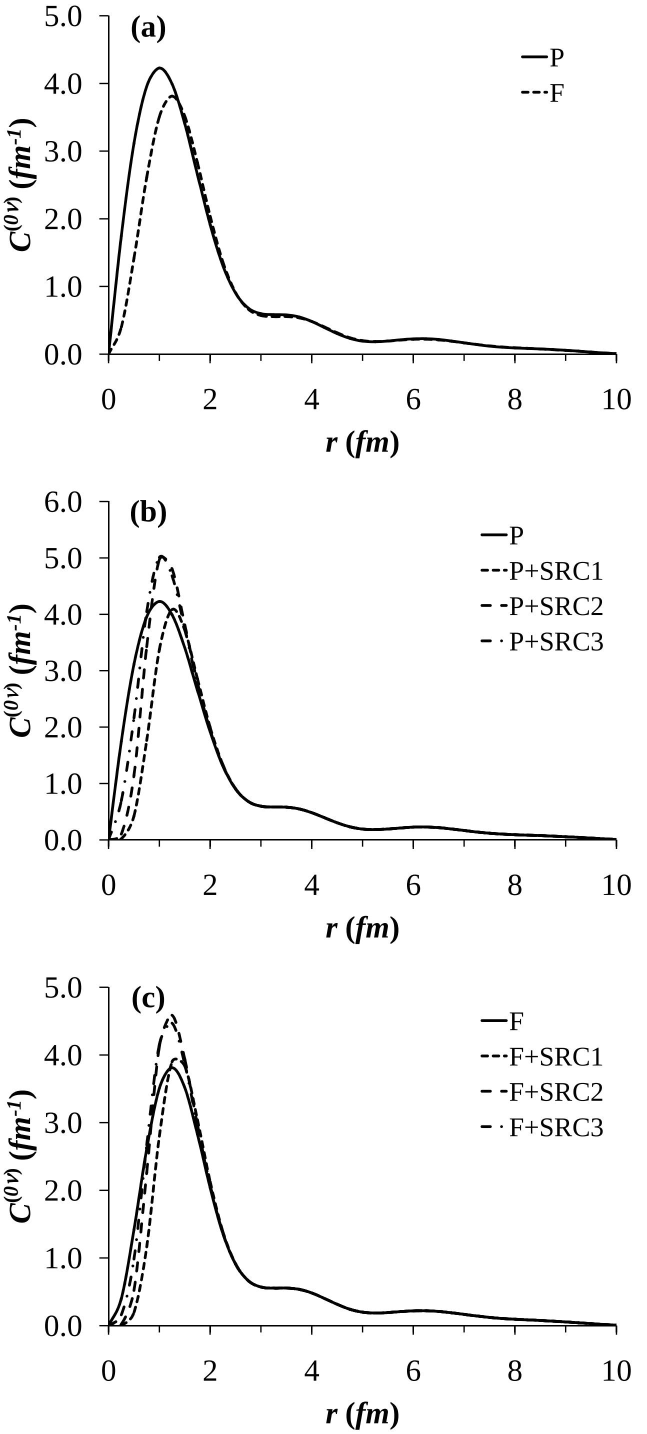

FIG. 3: Radial dependence of $C^{(0\nu)}(r)$ for the $\left(\beta^-\beta^-\right)_{0\nu}$ decay of ^{100}Mo isotope.

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